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NEUTROSOPHIC WEAKLY SEMI CONTINUOUS FUNCTIONS

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Abstract: In this paper, we introduce the concept of neutrosophic weakly semi continuous, neutrosophic regular semi q-neighbourhood in neutrosophic topological spaces. Moreover, we investigate the relationship among neutrosophic weakly semi continuous and other existing continuous functions and some counter examples to show that these types of mappings are not equivalent.

Keywords and Phrases: $NWSC$, NS-q-nbd, N-retracts, NS-retracts, NS-quasi Urysohn space, NS-Hausdorff space.

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1. Introduction

The study of fuzzy sets was initiated by Zadeh [23] in 1965. Thereafter the paper of Chang [4] paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Currently fuzzy topology has been observed to be very beneficial in fixing many realistic problems. Several mathematicians have tried almost all the pivotal concepts of General Topology for extension to the fuzzy settings. In 1983, Atanassov [1] introduced the concept of intuitionistic fuzzy set which was generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. Later, Coker [5] introduced the concept of intuitionistic topological spaces, by using the notion of the intuitionitic fuzzy set. Smarandache [16, 17, 18] introduced the concept of neutrosophic set. Neutrosophic set is classified into three independent functions namely, membership function, indeterminacy and non membership function that are independently related. In 2018, Smarandache and Pramanik [7, 19] introduced some concepts concerning the neutrosophic sets, single valued neutrosophic sets, interval-valued neutrosophic sets, bipolar neutrosophic sets, neutrosophic hesitant fuzzy sets, inter-valued neutrosophic hesitant fuzzy sets, refined neutrosophic sets, bipolar neutrosophic refined sets, multi-valued neutrosophic sets, simplified neutrosophic linguistic sets, neutrosophic over/off/under sets, rough neutrosophic sets, rough bipolar neutrosophic sets, rough neutrosophic hyper-complex set, and their basic operations. In this theory developed many researchers [2, 12, 13, 14, 15].

In 2012, Salama and Alblowi [9, 10, 11] introduced the concept of neutrosophic topology. Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allow more general functions to be members of fuzzy topology. In 2014, Salama et. al., [10] introduced the concept of neutrosophic closed sets and neutrosophic continuous functions.

In 2016, Iswarya and Bageerathi [6] introduced the concept of semiopen (semiclosed) sets, neutrosophic semi clousure (neutrosophic semi interior) in neutrosophic topological spaces. Recently, Bhimraj Basumatary et. al., [3] introduced neutrosophic semi continuous, neutrosophic semi open, neutrosophic semi closed, neutrosophic almost continuous mapping in neutrosophic topological spaces. In this paper, we introduce and study the concept of neutrosophic weakly semi continuous, neutrosophic semi q-neighbourhood in neutrosophic topological spaces. Moreover, we investigate the relationship among neutrosophic weakly semi continuous and other existing continuous functions and some counter examples to show that these types of mappings are not equivalent. Finally, neutrosophic retracts, neutrosophic semi retracts, neutrosophic quasi Urysohn space and neutrosophic semi Hausdorff spaces are introduced and studied.

2. Preliminary Definitions

In this section, we recollect some relevant basic preliminaries about neutrosophic sets and its operations.

Definition 2.1. [9] *Let X be a non-empty fixed set. A neutrosophic set [for short, Ns] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.*

Remark 2.2. [9] *A Ns $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple $A = \langle \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ in $]^-0, 1^+[$ on X.*

Remark 2.3. [9] *For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the Ns $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$.*

Example 2.4. [9] Every intuitionsistic fuzzy set A is a non-empty set in X is obviously on Ns having the form $A = \{\langle x, \mu_A(x), 1 - \mu_A(x) + \gamma_A(x)\rangle : x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic set and Neutrosophic topology, we must introduce the Neutrosophic sets 0_N and 1_N in X as follows:
$0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$.

Definition 2.5. [9] *Let $A = \langle(\mu_A, \sigma_A, \gamma_A)\rangle$ be a Ns on X, then the complement of the set A(A^c or $C(A)$ for short) may be defined as*
$C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$.

Definition 2.6. [9] *Let X be a non-empty set and Ns's A and B in the form $A = \{\langle x, \mu_A, \sigma_A, \gamma_A\rangle : x \in X\}$ and $B = \{\langle x, \mu_B, \sigma_B, \gamma_B\rangle : x \in X\}$. Then $(A \subseteq B)$ may defined as: $(A \subseteq B) \Leftrightarrow \mu_A(x) \subseteq \mu_B(x), \sigma_A(x) \subseteq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x) \forall x \in X$.*

Definition 2.7. [9] *Let X be a non-empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$ are Ns's. Then $A \cap B$ and $A \cup B$ may defined as:*

(I_1) $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle$

(U_1) $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle$

Definition 2.8. [9] *A Neutrosophic topology (for short, NT or nt) is a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:*

(i) $0_N, 1_N \in \tau_N$,

(ii) $G_1 \cap G_2 \in \tau_N$ *for any* $G_1, G_2 \in \tau_N$,

(iii) $\cup G_i \in \tau_N$ *for every* $\{G_i : i \in J\} \subseteq \tau_N$.

Throughout this paper, the pair of (X, τ_N) is called a neutrosophic topological space (for short, nts). The elements of τ_N or τ are called neutrosophic open set (for short, nos). A neutrosophic set F is neutrosophic closed (for short, ncs) if and only if F^c is nos.

Definition 2.9. [9] *Let (X, τ_N) be nts and $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$ be a Ns in X. Then the neutrosophic closure and neutrosophic interior of A are defined by $NCl(A) = \cap\{K : K$ is a NCS in X and $A \subseteq K\}$, $NInt(A) = \{G : G$ is a NOS in X and $G \subseteq A\}$. It can be also shown that $NCl(A)$ is NCS and $NInt(A)$ is a NOS in X. A is NOS if and only if $A = NInt(A)$, A is NCS if and only if $A = NCl(A)$.*

Definition 2.10. [6] *Let $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X\}$ be a Ns on a nts (X, τ_N) then A is called:*

(i) neutrosophic semiopen (for short, nso) if $A \subseteq NInt(NCl(A))$. Equivalently, neutrosophic semiopen if there exists an nos B in X such that $B \subseteq A \subseteq NCl(B)$.

(ii) neutrosophic semiclosed (for short, nsc) if there exists an ncs B in X and $NInt(B) \subseteq A \subseteq B$.

We shall denote the family of all nso sets (nsc sets) of a nts (X, τ) by $NSOS(X)$, $NSCS(X)$.

Definition 2.11. [6] *Let $(X, \ \tau)$ be a nts. Then*

(i) the neutrosophic semiclosure of A defined by $NSCl(A) = \bigcap\{B \mid A \subseteq B$ and $B \in NRSCS(X, \tau)\}$ is a neutrosophic set.

(ii) the neutrosophic semiinterior of A defined by $NSInt(A) = \bigcup\{B \mid B \subseteq A$ and $B \in nsoS(X, \tau)\}$ is a neutrosophic set.

Definition 2.12. *Let X be a nonempty set. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$, then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point (briefly NP) in X given by*

$$x_{r,t,st}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1) & \text{if } x \neq x_p. \end{cases}$$

for $x_p \in X$ is called the support of $x_{r,t,s}$, where r denotes the degree of membership value, t the degree of indeterminacy and s the degree of non-membership value of $x_{r,t,s}$.

Definition 2.13. [8] *Let A be a Ns over X, let $x_{\alpha,\beta,\gamma}$ be NP in X.*

(i) $x_{\alpha,\beta,\gamma}$ *is contained in* A *if* $\alpha \leq T_A(x)$, $\beta \geq I_A(x)$, $\gamma \geq F_A(x)$.

(ii) $x_{\alpha,\beta,\gamma}$ *is belong to* A *if* $\alpha \leq T_A(x)$, $\beta \geq I_A(x)$, $\gamma \geq F_A(x)$.

Definition 2.14. [10] *A NP* $x_{\alpha,\beta,\gamma} \in N(X)$ *is said to be quasi-coincident with a NS* $A \in N(X)$ *or* $x_{\alpha,\beta,\gamma} \in N(X)$ *quasi-coincides with a NS* $A \in N(X)$*, denoted by* $x_{\alpha,\beta,\gamma}qA$*, iff* $\alpha > T_{A^c}(x)$ *or* $\beta < I_{A^c}(x)$ *or* $\gamma < F_{A^c}(x)$*, i.e.,* $\alpha > F_A(x)$ *or* $\beta < 1 - I_A(x)$ *or* $\gamma < T_A(x)$*. A NS A is said to be quasi-coincident with a NS B at* $x \in X$ *or A quasi-coincides with B, denoted by AqB at x iff* $T_A(x) > T_{B^c}(x)$ *or* $I_A(x) < I_{B^c}(x)$ *or* $F_A(x) < F_{B^c}(x)$.

Definition 2.15. [8] *A Ns A is called an neutrosophic quasi-neighbourhood (for short, N-q-nbd) of a neutrosophic point* $x_{\alpha,\beta,\gamma}$ *iff there exists a NS* $B \in \tau$ *such that* $x_{r,t,s}qB \subseteq A$.

Theorem 2.16. *For any Ns A in an nts* (X,τ), $NInt(A) \subseteq NSInt(A) \subseteq A \subseteq NSCl(A) \subseteq NCl(A)$.

Theorem 2.17. [11] *Let* $f : X \to Y$ *a function. Then*

(i) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$,

(ii) $B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(iii) $A \subseteq f^{-1}(f(A))$ *and if f is injective, then* $A = f^{-1}(f(A))$,

(iv) $f^{-1}(f(B)) \subseteq B$ *and if f is surjective, then* $f^{-1}(f(B)) = B$,

(v) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$,

(vi) $f(\cup A_i) = \cup f(A_i)$, $f(\cap A_i) \subseteq \cap f(A_i)$ *and if f is injective, then* $f(\cap A_i) = \cap f(A_i)$,

Definition 2.18. [11] *Let* (X,τ) *and* (Y,σ) *be any two nts's. A map* $f : (X,\tau) \to (Y,\sigma)$ *is neutrosophic continuous if the inverse image of every neutrosophic closed set in* (Y,σ) *is neutrosophic closed set in* (X,τ).

Definition 2.19. [3] *Let* (X,τ) *and* (Y,σ) *be any two nts's. A map* $f : (X,\tau) \to (Y,\sigma)$ *is*

(i) neutrosophic semi continuous (for short, NSC) if the inverse image of every nos in (Y,σ) *is nso set in* (X,τ).

(ii) neutrosophic semi irresolute (for short, NSI) if the inverse image of every nso set in (Y,σ) *is nso set in* (X,τ).

Theorem 2.20. *If* $f : X \to Y$ *is NSC and NAO, then f is NSI.*

3. Neutrosophic weakly semi continuous functions

Definition 3.1. *A mapping $f : X \to Y$ is called the :*

(i) neutrosophic weakly continuous (for short, NWC) iff for any NP $x_{r,t,s}$ in X and any nos B of Y containing $f(x_{r,t,s})$, there exists an nos A containing $x_{r,t,s}$ such that $f(A) \subseteq NCl(B)$.

(ii) neutrosophic weakly semi continuous (for short, $NWSC$) iff for any NP $x_{r,t,s}$ in X and any nos B of Y containing $f(x_{r,t,s})$, there exists an nso set A containing $x_{r,t,s}$ such that $f(A) \subseteq NSCl(B)$.

Example 3.2. Let $X = \{a, b\}$ and $\tau = \{0_N, 1_N, X, A, B\}$, $Y = \{p, q\}$ and $\sigma = \{0_N, 1_N, C\}$, where A and B are Ns of X and C is Ns of Y, defined as follows:

$$A = \left\langle \left(\tfrac{\mu_a}{0.3}, \tfrac{\mu_b}{0.5}\right), \left(\tfrac{\sigma_a}{0.5}, \tfrac{\sigma_a}{0.5}\right), \left(\tfrac{\gamma_a}{0.6}, \tfrac{\gamma_b}{0.5}\right) \right\rangle,$$
$$B = \left\langle \left(\tfrac{\mu_a}{0.6}, \tfrac{\mu_b}{0.5}\right), \left(\tfrac{\sigma_a}{0.5}, \tfrac{\sigma_a}{0.5}\right), \left(\tfrac{\gamma_a}{0.5}, \tfrac{\gamma_b}{0.5}\right) \right\rangle,$$
$$C = \left\langle \left(\tfrac{\mu_p}{0.4}, \tfrac{\mu_q}{0.5}\right), \left(\tfrac{\sigma_p}{0.5}, \tfrac{\sigma_q}{0.5}\right), \left(\tfrac{\gamma_p}{0.6}, \tfrac{\gamma_q}{0.5}\right) \right\rangle.$$

Clearly τ and σ are NT on X and Y. If we define the function $f : X \to Y$ as $f(a) = p$ and $f(b) = q$, then f is $NWSC$ but not NWC, for any NP $x_{0.4,0.5,0.6}$ in X and a nos C of Y containing $f(x_{0.5,0.6,0.6})$, there exists a nso $D = \left\langle \left(\frac{\mu_p}{0.4}, \frac{\mu_q}{0.5}\right), \left(\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}\right), \left(\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.5}\right) \right\rangle$ [D is nso set of X, since $\exists$ a nos B such that $B \subseteq D \subseteq NCl(B)$] containing $x_{0.4,0.5,0.6}$ such that $f(D) \subseteq NSCl(C)$. But D is not nos.

Remark 3.3. *The above Definition and Examples 3.2, it clear that every NWC functions is $NWSC$ but not conversely.*

Definition 3.4. *An nts (X, τ) is N-regular iff for each NP $x_{r,t,s}$ (for short NP)in X and each N-open-q-nbd A of $x_{r,t,s}$, there exists N-open-q-nbd B of $x_{r,t,s}$ such that $NCl(B) \subseteq A$.*

Theorem 3.5. *If Y is an N-regular space, then a mapping $f : X \to Y$ is $NWSC$ iff f is NSC.*

Proof. The necessary part follows from Remark 3.3. We prove only the sufficient part. Let f be $NWSC$ and Y be an N-regular space. Let $x_{r,t,s}$ be any NP of X and B be any nos in Y containing $f(x_{r,t,s})$. Since Y is N-regular, there exists an N-open-q-nbd C of $f(x_{r,t,s}) = Y_{r,t,s}$ (where $y = f(x)$) such that $NSCl(C) \subseteq B$. Since f is $NWSC$ and C is an N-open-q-nbd of $f(x_{r,t,s})$, there exists $A \in NSOS(X)$ with $x_{r,t,s} \in A$ such that $f(A) \subseteq NSCl(C)$. By Theorem 2.20, $NSCl(C) \subseteq NCl(C)$ and so $f(A) \subseteq NSCl(C) \subseteq NCl(C) \subseteq B$. Thus f is NSC by Definition 3.1 and this completes the proof.

In the following theorems we give some characterization of $NWSC$ functions.

Theorem 3.6. *A mapping $f : X \to Y$ is $NWSC$ iff for each nos B in Y, $f^{-1}(B) \subseteq NSInt(f^{-1}(NSCl(B)))$.*
Proof. Let f be $NWSC$ and B be any nos set in Y. Let $x_{r,t,s}$ be an NP in $f^{-1}(B)$. Thus $f(x_{r,t,s}) \in B$, f is $NWSC$ implies that there exists an $A \in NSOS(X)$ such that $x_{r,t,s} \in A$ and $f(A) \subseteq NSCl(B)$. By Theorem 2.17 (2) and (3) we have $A \subseteq f^{-1}(NSCl(B))$. Hence $NSInt(A) \subseteq NSInt(f^{-1}(NSCl(B)))$ and since A is nso, $A \subseteq NSInt(f^{-1}(NSCl(B)))$. So $f^{-1}(B) \subseteq A \subseteq NSInt(f^{-1}(NSCl(B)))$.

Conversely let $x_{r,t,s}$ be an NP in X and B be any nos set in Y such that $f(x_{r,t,s}) \in B$. By hypothesis, $f^{-1}(B) \subseteq NSInt(f^{-1}(NSCl(B)) = A$(say). Hence $x_{r,t,s} \in f^{-1}(B) \subseteq A$, which implies that A is an nso set in X containing $x_{r,t,s}$. So $A = NSInt(f^{-1}(NSCl(B))) \subseteq f^{-1}(NSCl(B)$, i.e., $f(A) \subseteq NSCl(B)$ (by Theorem 2.17(4)). Hence f is $NWSC$ and this proves the result.

Theorem 3.7. *A mapping $f : X \to Y$ is $NWSC$ if for each nos B in Y, $f^{-1}(NSCl(B)) \in NSOS(X)$.*
Proof. Straightforward.

Theorem 3.8. *If $f : X \to Y$ is neutrosophic irresolute and $g : Y \to Z$ is $NWSC$, then $g \circ f : X \to Z$ is $NWSC$.*
Proof. Let $x_{r,t,s}$ be an NP in X and C be any nos in Z containing $((g \circ f)(x_{r,t,s})) = g(f(x_{r,t,s}))$. Since g is $NWSC$ there exists an nos B in Y containing $f(x_{r,t,s})$ such that $g(B) \subseteq NSCl(C)$. Also since f is neutrosophic irresolute and B is nos in Y, it follows that $f^{-1}(B)$ is nso in X. Let $A = f^{-1}(B)$. Now $(g \circ f)(A) = g(f(A)) \subseteq g(B) \subseteq NSCl(C)$. So $g \circ f$ is $NWSC$ and this completes the proof.

Corollary 3.9. *If $f : X \to Y$ is NSC and $NAOorNO$-function and $g : Y \to Z$ is $NWSC$, then $g \circ f$ is $NWSC$.*
Proof. The proof follows from Theorems 2.20 and 3.8.

4. Neutrosophic weakly semi continuous in terms of q-coincidence, q-neighborhoods and θ-cluster points

Definition 4.1. *A Ns A is called an neutrosophic semi-q-nbd (for short, NS-q-nbd) of an NP $x_{r,t,s}$ in an nts (X, τ) iff there exists an nso set B of X such that $x_{r,t,s} qB \subseteq A$.*

Definition 4.2. *A Ns A in an nts (X, τ) is said to be an neutrosophic θ-nbd (for short, N-θ-nbd), (neutrosophic semi θ-nbd (for short, NS-θ-nbd)) of an NP $x_{r,t,s}$ iff there exists an N-closed-q-nbd (NS-closed-q-nbd) B of $x_{r,t,s}$ such that $B\overline{q}A^c$, i.e., $B \subseteq A$.*

Definition 4.3. *A NP $x_{r,t,s}$ in a nts (X, τ) is called an neutrosophic semi clus-*

ter point (for short, NS-cluster point) of an Ns A iff every NS-q-nbd of $x_{r,t,s}$ is q-coincident with A. The set of all NS-cluster points of an Ns A is called neutrosophic semi closure of A and denoted by $NSC(A)$.

Definition 4.4. *An NP $x_{r,t,s}$ in a nts (X, τ) is called on neutrosophic semi θ-cluster point (for short, NS-θ-cluster point)of an Ns A iff every NS-open-q-nbd B of $x_{r,t,s}$, $NSCl(B)$ is q-coincident with A. The set of all NS-θ-cluster points of an Ns A is called neutrosophic semi θ closure of A and it is denoted by $NS\theta Cl(A)$.*

Theorem 4.5. *A mapping $f : X \to Y$ is $NWSC$ iff corresponding to each N-open-q-nbd B of $y_{r,t,s}$ in Y, there exists an NS-open-q-nbd A of $x_{r,t,s}$ in X such that $f(A) \subseteq NSCl(B)$, where $f(x_{r,t,s}) = (f(x))_{r,t,s} = y_{r,t,s}$.*

Proof. Let f be $NWSC$ and B be an N-open-q-nbd of $y_{r,t,s}$, where $f(x) = y$ in Y. So, $B(y) + (r, t, s) > 1_N$. We can choose a positive real number (u, v, w) such that $B(y) > (u, v, w) > 1 - (r, t, s)$. Hence B is an N-open-nbd of $y_{u,v,w}$ in Y. Since f is $NWSC$, there exists an nso set A containing $x_{u,v,w}$ such that $f(A) \subseteq NSCl(B)$. Now $A(x) \geq (u, v, w)$ implies $A(x) > 1 - (r, t, s)$, i.e., $A(x) + (r, t, s) > 1$. Thus $x_{r,t,s}qA$. So A is an NS-open-q-nbd of $x_{r,t,s}$.

Conversely, let the condition of the theorem hold, i.e., let $x_{r,t,s}$ be an NP in X and B be an nos in Y containing $y_{r,t,s} = (f(x))_{r,t,s}$. So $x_{r,t,s} \in f^{-1}(B) = C$ (say). Hence $C(x) \geq (r, t, s)$. We can choose a (u, v, w) such that $C(x) \geq 1/(u, v, w)$. Put $(r, t, s)_n = 1 + (1/n) - C(x)$, for any positive integer $n \geq (u, v, w)$. Clearly $0 < (r, t, s)_n \subseteq 1$ for all $n \geq (u, v, w)$. Now $B(y) + (r, t, s)_n = B(y) + 1 + (1/n) - C(x) = 1 + (1/n) > 1$ (Since $C(x) = f^{-1}(B)(x) = B(f(x)) = B(y)$). Hence $y_{r,t,s_n}qB$, i.e., B is an N-open-q-nbd of y_{r,t,s_n} for all $n \geq (u, v, w)$. So by hypothesis there exists an NS-open-q-nbd A_n of x_{r,t,s_n} such that $f(A_n) \subseteq NSCl(B)$, for all $n \geq (u, v, w)$. Now $A = \cup_{n \geq (u,v,w)} A_n$ is nso in X. It remains to show that $x_{r,t,s} \in A$. We have $A_n(X) > 1 - (r, t, s)_n = C(x) - (1/n)$ for all $n \geq (u, v, w)$. Thus $A(x) > C(x) - (1/n)$ for all $n \geq (u, v, w)$. Since $x_{r,t,s} \in C$, $A(x) \geq C(x) \geq (r, t, s)$. So A is an nso set in X such that $f(A) = f(\cup_{n \geq (u,v,w)} A_n) = \cup_{n \geq (u,v,w)} f(A_n) \subseteq NSCl(B)$. Hence f is $NWSC$ and this completes the proof.

Lemma 4.6. *For any two Ns's A and B in X, $A \subseteq B$ iff for each $x_{r,t,s}$ in X, $x_{r,t,s} \in A$ then $x_{r,t,s} \in B$.*

Lemma 4.7. *Let $f : X \to Y$ be any neutrosophic function and $x_{r,t,s}$ be any NP in X, then*

(i) for $A \subseteq X$ and $x_{r,t,s}qA$, we have $f(x_{r,t,s})qf(A)$.

(ii) for $B \subseteq Y$ and $f(x_{r,t,s})qB$, we have $x_{r,t,s}qf^{-1}(B)$.

Theorem 4.8. *If $f : X \to Y$ is an $NWSC$, then for each nos B in Y, $NSCl(f^{-1}(B) \subseteq f^{-1}(NSCl(B)$.*

Proof. Suppose that there is an NP $x_{r,t,s} \in NSCl(f^{-1}(B))$ but $x_{r,t,s} \notin f^{-1}(NCl(B))$. Since $f(x_{r,t,s}) \notin NCl(B)$ there exists an nos C in Y with $f(x_{r,t,s}) \in C$ such that $C\overline{q}NCl(B)$. Thus $C\overline{q}B$ and $NCl(C)\overline{q}B$. Since f is $NWSC$, there exists an $A \in FSOS(X)$ with $x_{r,t,s} \in A$ such that $f(A) \subseteq NSCl(C)$. Hence $f(A)\overline{q}B$, since $f(A) \subseteq NSCl(C) \subseteq NCl(C)$. But on the other hand, since each nso set is NS-q-nbd of each of its NP $x_{r,t,s} \in NSCl(f^{-1}(B))$ and A is an NS-q-nbd of $x_{r,t,s}$. By Definition 4.3, $Aqf^{-1}(B)$. By Lemma 4.7(1), $f(A)qf(f^{-1}(B))$, and hence by theorem 2.17(4), $f(A)qB$ which is a contradiction. This completes the proof of the theorem.

Theorem 4.9. *Let $f : X \to Y$ be an NO-mapping and $NWSC$ mapping. Then $f(NSCl(A) \subseteq NCl(f(A))$, for each nos A in X.*

Proof. Let A be an nos set in X and let $f(A) = B$. Since f is NO, we see that B is an nos in Y. Hence by Theorem 2.17(3), $A \subseteq f^{-1}(f(A)) = f^{-1}(B)$. Since f is $NWSC$, we have from Theorem 4.8, $NSCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B)$. Thus $NSCl(A) \subseteq f^{-1}(NCl(B)$, i.e., $f(NSCl(A)) \subseteq NCl(B) = NCl(f(A))$ and this proves the result.

Theorem 4.10. *A function $f : X \to Y$ is $NWSC$ iff for each nos B in Y, $(x_{r,t,s})qf^{-1}(B)$ implies $(x_{r,t,s})qf^{-1}(NSInt(NSCl(B))$ for each NP $x_{r,t,s}$ in X.*

Proof. Let f is $NWSC$. Let $x_{r,t,s}$ be NP in X and B be any nos in Y such that $(x_{r,t,s})qf^{-1}(B)$. Then $f(x_{r,t,s})qB$. Since f is $NWSC$ by Theorem 4.5, there exists an nso set A in X such that $(x_{r,t,s})qA$ and $f(A) \subseteq NSCl(B)$. By Theorem 2.17(2) and (3) we have $A \subseteq f^{-1}(NSCl(B))$. Since $A \in NSOS(X)$, $A \subseteq f^{-1}(NSInt(NSC(B))$. So $(x_{r,t,s})qf^{-1}(NSInt(NSCl(B))$.

Conversely, let the condition given in the statement hold. Let $x_{r,t,s}$ be an NP in X and B be an N-open-q-nbd of $f(x_{r,t,s})$ such that $(x_{r,t,s})qf^{-1}(B)$. By hypothesis, $(x_{r,t,s})qf^{-1}(NSInt(NSCl(B))$. Put $A = f^{-1}(NSInt(NSCl(B))$. Hence $A \in NSOS(X)$. $(x_{r,t,s})qA$ implies that A is an NS-open-q-nbd of $x_{r,t,s}$. Also $f(f^{-1}(NSInt(NSCl(B)))) \subseteq f(f^{-1}(NSCl(B)))$, i.e., $f(A) \subseteq NSCl(B)$. Thus f is $NWSC$ and this completes the proof.

Theorem 4.11. *If $f : X \to Y$ is $NWSC$ then for each NP $x_{r,t,s}$ in X and each N-θ-nbd B of $f(x_{r,t,s})$, $f^{-1}(B)$ is an NS-q-nbd of $x_{r,t,s}$.*

Proof. Let $f : X \to Y$ is $NWSC$ function and $x_{r,t,s}$ be an NP in X. Let $NSCl(B)$ be an N-θ-nbd of $f(x_{r,t,s})$. So there is an N-open-q-nbd C of $f(x_{r,t,s})$ such that $NCl(C)\overline{q}B^c$, i.e., $NCl(C) \subseteq B$. Since C is an N-open-q-nbd of $f(x_{r,t,s})$, by Theorem 4.5, there is an NS-open-q-nbd A of $x_{r,t,s}$ such that $f(A) \subseteq NSCl(C)$ and thus $f(A) \subseteq NSCl(C) \subseteq NCl(C) \subseteq B$. So $A \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is an NS-q-nbd of $x_{r,t,s}$ and this completes the proof.

Theorem 4.12. *If $f : X \to Y$ is a function such that for each NP $x_{r,t,s}$ in X and each NS-θ-nbd B of $f(x_{r,t,s})$ in Y, $f^{-1}(B)$ is NS-q-nbd of $x_{r,t,s}$ in X, then f is $NWSC$.*

Proof. Let $x_{r,t,s}$ be an NP in X and B be any N-open-q-nbd of $f(x_{r,t,s})$. We note that B is an NS-open-q-nbd of $f(x_{r,t,s})$. So $NSCl(B)$ is an NS-θ-nbd of $f(x_{r,t,s})$. By hypothesis, $f^{-1}(NSCl(B))$ is an NS-q-nbd of $x_{r,t,s}$. So there exists an nso set A in X such that $x_{r,t,s}qA \subseteq f^{-1}(NSCl(B))$, i.e., $f(A) \subseteq NSCl(B)$. Hence f is $NWSC$ and this completes the proof.

Theorem 4.13. *If $f : X \to Y$ is $NWSC$, then*

(i) $f(NSCl(A) \subseteq NS\theta Cl(f(A))$ for each Ns A in X,

(ii) $f(NSCl(f^{-1}(NSCl(NSInt(B))))) \subseteq NS\theta Cl(B)$ for each Ns B in Y.

Proof. (i) Let $x_{r,t,s} \in NSCl(A)$ and S be an N-closed-q-nbd of $f(x_{r,t,s})$. Then there exists an N-open-q-nbd V of $f(x_{r,t,s})$ such that $V \subseteq S$. Since f is $NWSC$, by Theorem 4.5 there exists an NS-open-q-nbd U of $x_{r,t,s}$ such that $f(U) \subseteq NSCl(V)$. Since $x_{r,t,s} \in NSCl(A)$, by Definition 4.3, $x_{r,t,s}$ is an NS-cluster point of A. Hence UqA and also $f(U)qf(A)$. Since $f(U) \subseteq NSCl(V)$, $NSCl(V)qf(A)$. Again since, S is ncs, we have $NSCl(V) \subseteq NCl(V) \subseteq S$. Therefore $Sqf(A)$. By Definition 4.4, $f(x_{r,t,s}) \in NS\theta Cl(f(A))$, i.e., $x_{r,t,s} \in f^{-1}(NS\theta Cl(f(A))$. Therefore, $NSCl(A) \subseteq f^{-1}(NS\theta Cl(f(A)))$; which implies $f(NSCl(A)) \subseteq NS\theta Cl(f(A))$, proving (i).

(ii) Let B be an Ns in Y and $x_{r,t,s}$ be an NP in X such that $x_{r,t,s} \in NSCl(f^{-1}$ $(NSCl(NSInt\ (B))))$. Let V be any N-open-q-nbd of $f(x_{r,t,s})$. By Theorem 4.5, there exists NS-open-q-nbd U of $x_{r,t,s}$ such that $f(U) \subseteq NSCl(V)$. Since $NSCl(f^{-1}(NSInt(B))) \subseteq NSCl(f^{-1}(B))$, we have $x_{r,t,s} \in NSCl\ (f^{-1}\ (NSCl$ $(NSInt(B))))$. By Definition 4.4, $Uqf^{-1}(B)$, i.e.,$f(U)qB$. Thus $NSCl(V)qB$, which implies $f(x_{r,t,s}) \in NS\theta Cl(B)$. So $f(NSCl(f^{-1}(NSCl(NSInt(B))))) \subseteq NS\theta Cl(B)$, proving (ii).

5. Neutrosophic weakly regular semi continuous functions and neutrosophic retracts

Definition 5.1. *Let X ba a nts and $A \subseteq X$. Then the subspace (crisp) A of X is called a neutrosophic retract (for short, N-retract) of X if there exists a neutrosophic continuous function $r : X \to A$ such that $r(a) = a$ for all $a \in A$. In this case r is called a N-retraction. A is called an NO-retract (NSC-retract) of X if r is N-open (NSC). Similarly, A is called $NWSC$-retract of X if r is $NWSC$.*

Theorem 5.2. *If A is an NO-retract, NSC-retract of the nts X then for every nts Y, any $NWSC$ function $g : A \to Y$ can be extended to an $NWSC$ function of X into Y.*

Proof. Let Y be an arbitrary nts and $g : A \to Y$ be an $NWSC$ function. By Corollary 3.9, $g \circ r : X \to Y$ is $NWSC$ and $g \circ r(a) = g(r(a)) = g(a)$ for all $a \in A$, where $r : X \to A$ is a NO-retract, NSC-retraction. Hence $g \circ r$ is an $NWSC$ extension of g to X and this completes the proof.

Theorem 5.3. *If A is an NO-retract, NSC-retract of X and B is an $NWSC$-retract of A then B is an $NWSC$-retract of X.*

Proof. Let $r : X \to A$ be an N-open and $NRSC$ mapping such that $r(a) = a$ for all $a \in A$. Let $s : A \to B$ be an $NWSC$ retraction of A such that $s(b) = b$ for all $b \in B$. By corollary 3.9, $s \circ r : X \to B$ is $NWSC$ and $s \circ r(b) = b$ for all $b \in B$. Hence B is $NWSC$-retract of X and this proves the result.

Theorem 5.4. *An nts X is called an N-quasi Urysohn space if for any two distinct NP's $x_{r,t,s}$ and $y_{r,t,s}$, there exist nos's U_1 and U_2 in X such that $x_{r,t,s}qU_1$, $y_{r,t,s}qU_2$ and $NCl(U_1) \cap NCl(U_2) = 0_N$.*

Theorem 5.5. *An nts X is said to be NS-quasi Hausdorff if distinct NP's in X have disjoint NS-q-nbds, i.e., if $x_{r,t,s}$ and $y_{r,t,s}$ are distinct NP's in X, then there exist NS-q-nbds V_1 and V_2 such that $x_{r,t,s}qV_1$, $y_{r,t,s}qV_2$ and $V_1 \cap V_2 = 0_N$.*

Theorem 5.6. *If Y is an N-quasi Urysohn space and $f : X \to Y$ is an $NWSC$ injection, then X is a NS-quasi Hausdorff space.*

Proof. Let $x_{r,t,s}$ and $y_{r,t,s}$ be two distinct NP's in X. f being injective, $f(x_{r,t,s})$ and $f(y_{r,t,s})$ are distinct NP's in Y. Since Y is N-quasi Urysohn, there exists nos's V_1 and V_2 in Y such that $f(x_{r,t,s})qV_1$, $f(y_{r,t,s}qV_2)$ and $NCl(V_1) \cap NCl(V_2) = 0_N$, i.e., $f^{-1}(NSInt(NCl(V_1)) \cap f^{-1}(NSInt(NCl(V_2)) = 0_N$, By Theorem 3.6, $x_{r,t,s}qf^{-1}(V_1) \subseteq f^{-1}(NSInt(NSCl(V_1))) \subseteq f^{-1}(NSInt(NCl(V_1))$. Similarly $y_{r,t,s}q$ $f^{-1}(V_2) \subseteq f^{-1}(NSInt(NSCl(V_2))) \subseteq f^{-1}(NSInt\ (NCl(V_2))$. So, $f^{-1}(NSInt$ $(NCl(V_1))$ and $f^{-1}(NInt(NCl(V_2))$ are disjoint NS-q-nbds of $x_{r,t,s}$ and $y_{r,t,s}$, respectively. So X is NS-quasi Hausdorff and this proves the result.

6. Conclusion

The basic aim of this paper we introduced the concept of neutrosophic weakly semi continuous, neutrosophic regular semi q-neighbourhood in neutrosophic topological spaces. Moreover, the relationship among neutrosophic weakly semi continuous and other existing continuous functions and some counter examples to show that these types of mappings are not equivalent. Finally, Neutrosophic retracts, neutrosophic semi retracts, neutrosophic quasi Urysohn space and neutrosophic semi Hausdorff spaces are introduced and studied. In future, we promote this thought into neutrosophic semi β continuous mappings, neutrosophic contra semi β continuous and contra semi β irresolute mappings in neutrosophic topological spaces. Further, we work may include the extension of this work for NeutroTopology and AntiTopology which got some attention from researchers.

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References

[1] Atanassov, K. T., Intuitionistic neutrosophic sets, neutrosophicSets and Systems, 20 (1986), 87-96.

[2] Balamurugan, P., Sudha, S. and Gnanam, A., Establishment of even order a subsequence of a farey sequence as a topological space, Advances and Applications in Mathematical Sciences, 21 (7) (2020), 3895-3904.

[3] Bhimraj B., Nijwm W., Jeevan K. K. and Usha R. B., On Some Properties of Neutrosophic Semi Continuous and Almost Continuous Mapping, Computer Modeling in Engineering and Sciences, 130, (2) (2022), 1117-1131.

[4] Chang, C. L, neutrosophic topological spaces, J. Math. Anal. Appl., 24 (1968), 182-190.

[5] Coker D., An introduction to intuitionistic neutrosophic topological spaces, neutrosophic Sets and Systems, 88 (1997), 81-89.

[6] Ishwarya P. and Bageerathi K., On Neutrosophic semi-open sets in Neutrosophic topological spaces, International Jour. of Math. Trends and Tech., (2016), 214-223.

[7] Pramanik, S., Single-Valued Neutrosophic Set: An Overview. In: Rezaei, N. (eds) Transdisciplinarity, Integrated Science, Vol 5 (2022), 563-608.

[8] Ray, G. C. and Dey, S., Relation of Quasi-coincidence for Neutrosophic Sets, Neutrosophic sets and systems, 46 (2021), 402-415.

[9] Salama, A. A. and Alblowi S. A., Neutrosophic set and Neutrosophic topological space, ISOR J. Mathematics, 3(4) (2012), 31-35.

[10] Salama, A. A. and Alblowi, S. A., Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, Journal computer Sci. Engineering, 2(7) (2012), 12-23.

[11] Salama, A. A., Smarandache, F. and Valeri K., Neutrosophic Closed set and Neutrosophic Continuous Function, Neutrosophic Sets and Systems, 4 (2014), 4-8.

[12] Silambarasan, I., A note on n-HyperSpherical Neutrosophic matrices, Neutrosophic set and systems, 55 (2023), 344-357.

[13] Silambarasan, I., Some Algebraic Structures of Neutrosophic fuzzy sets, IJNS, 19(2) (2022), 30-41.

[14] Silambarasan, I., New operations defined over the q-Rung Orthopair fuzzy sets, Journal of the international mathematical virtual institute, 10(2) (2020), 341-359.

[15] Silambarasan, I., New operators for Fermatean fuzzy sets, Annals of Communications in Mathematics, 3(2) (2020), 116-131.

[16] Smarandache, F., A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic set, Neutrosophic Probability, American Research Press, Rehoboth, NM, 1999.

[17] Smarandache, F., Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA, 2002.

[18] Smarandache, F., Neutrosophic Set: A Generalization of Intuitionistiic neutrosophicset, Journal of Defense Resources Management, 1 (2010), 107-116.

[19] Smarandache, F., and Pramanik, S., Neutrosophic sets: An overview, New trends in neutrosophic theory and applications, Eds., Vol. 2 (2018), 403-434.

[20] Vijayalakshmi, R., and Praveena, R. R., Regular semiopen sets in neutrosophic topological spaces, Indian Journal of Natural Sciences, 12(70) (2022), 38114-38118.

[21] Vijayalakshmi, R., and Praveena, R. R., Neutrosophic Regular semi continuous functions, Annals of Communications in Mathematics, 4(3) (2021), 254-260.

[22] Wadel F. Al-omeri and Smarandache, F., New Neutrosophic Sets via Neutrosophic Topological Spaces, New Trends in Neutrosophic Theory and Applications, 2, (2016).

[23] Zadeh, L. A., Neutrosophicset, Inform and Control, 8 (1965), 338-353.